

# Can Massless and Light Yukawaons be Harmless?

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## Abstract

In the so-called yukawaon model, where the effective Yukawa coupling constants  $Y_f^{eff}$  ( $f = e, \nu, u, d$ ) are given by vacuum expectation values (VEVs) of gauge singlet scalars (yukawaons)  $Y_f$  with  $3 \times 3$  components, i.e.  $Y_f^{eff} = y_f \langle Y_f \rangle / \Lambda$ , massless (and light) scalars appear because a global flavor symmetry is assumed. In order to demonstrate whether such massless scalars in the yukawaon model are harmless or not, yukawaon masses are explicitly estimated, as an example, in the charged lepton sector.

## 1 Introduction

In the standard model of the quarks and leptons, the mass spectra and mixings originate in structures of the Yukawa coupling constants  $Y_f$  ( $f = u, d, \nu, e$ ), which are fundamental constants in the theory. In order to reduce the number of such the fundamental constants, usually, we assume flavor symmetries, and thereby, we discuss relations among the mass spectra and mixings. However, even if we assume such symmetries, some of such Yukawa coupling constants still remain as the fundamental constants in the theory. On the other hand, there is another idea for the origin of the mass spectra and mixings which we refer as a yukawaon model [1, 2]: We regard the Yukawa coupling constants  $Y_f$  as “effective” coupling constants  $Y_f^{eff}$  in an effective theory, and we consider that  $Y_f^{eff}$  originate in vacuum expectation values (VEVs) of new gauge singlet scalars  $Y_f$ , i.e.

$$Y_f^{eff} = \frac{y_f}{\Lambda} \langle Y_f \rangle, \quad (1.1)$$

where  $\Lambda$  is a scale of the effective theory. We refer the fields  $Y_f$  as “yukawaons”. In the yukawaon model, we can, in principle, calculate the VEVs  $\langle Y_f \rangle$  from a superpotential which is given in the model (although it is not yet established at present). In the yukawaon model, Higgs scalars are the same as those in the conventional model, i.e. we consider only two Higgs scalars  $H_u$  and  $H_d$  as the origin of the masses (not as the origin of the mass spectra). We also consider that the scale  $\Lambda$  is considerably large (for example,  $\Lambda \sim 10^{12}$  GeV). Therefore, we can inherit successful results from the standard model conveniently, and we can introduce any flavor symmetries without troubles with the gauge  $SU(2)_L$  symmetry (a trouble with  $SU(2)_L$ , for example, see Ref.[3, 4]).

As a model which describes masses and mixings by patterns of VEV values of one or more scalars, the Froggatt-Nielsen model [5] is well known: The hierarchical structure of the masses is explained by a multiplicative structure  $(\langle \phi \rangle / \Lambda)^n$  under a  $U(1)$  flavor symmetry. In contrast to the Froggatt-Nielsen model, in the yukawaon model, the hierarchical structure of the quark and lepton masses is understood from hierarchical eigenvalues of  $\langle Y_f \rangle$ , not from the multiplicative structure  $(\langle Y_f \rangle / \Lambda)^n$ . In a supersymmetric (SUSY) yukawaon model, the VEVs of yukawaons  $\langle Y_f \rangle$  are obtained from SUSY vacuum conditions for a superpotential  $W$ , so that the

supersymmetry is unbroken at  $\mu \sim \Lambda$ . We will assume that the superpotential  $W$  is invariant under a flavor symmetry. On the other hand, in order to distinguish a yukawaon  $Y_f$  from another yukawaons  $Y_{f'}$ , we will assume a  $U(1)_X$  symmetry: For example, we assume an  $O(3)$  flavor symmetry [6] and we consider that the yukawaons  $Y_f$  are  $(\mathbf{3} \times \mathbf{3})_S = \mathbf{1} + \mathbf{5}$  of  $O(3)_F$ . Then, the would-be Yukawa interactions are given by

$$H_Y = \sum_{i,j} \frac{y_u}{\Lambda} u_i^c (Y_u)_{ij} q_j H_u + \sum_{i,j} \frac{y_d}{\Lambda} d_i^c (Y_d)_{ij} q_j H_d \\ + \sum_{i,j} \frac{y_\nu}{\Lambda} \ell_i (Y_\nu)_{ij} \nu_j^c H_u + \sum_{i,j} \frac{y_e}{\Lambda} \ell_i (Y_e)_{ij} e_j^c H_d + h.c. + \sum_{i,j} y_R \nu_i^c (Y_R)_{ij} \nu_j^c, \quad (1.2)$$

where  $q$  and  $\ell$  are  $SU(2)_L$  doublet fields, and  $f^c$  ( $f = u, d, e, \nu$ ) are  $SU(2)_L$  singlet fields. Here, in order to distinguish each  $Y_f$  from others, we have assigned  $U(1)_X$  charges as  $Q_X(f^c) = -x_f$ ,  $Q_X(Y_f) = +x_f$  and  $Q_X(Y_R) = 2x_\nu$ . The  $SU(2)_L$  doublet fields  $q$ ,  $\ell$ ,  $H_u$  and  $H_d$  have sector charges  $Q_X = 0$ . As a result of requiring SUSY vacuum conditions, cross terms among yukawaons  $Y_f$  in different sectors  $f = u, d, e, \nu$  are allowed in the superpotential  $W$ . Therefore, in the yukawaon model, a mass matrix  $M_f$  in a sector  $f$  can be expressed in terms of mass matrices in another sectors. For example, we can build a model in which the Majorana mass matrix  $Y_R$  of the right-handed neutrino is related to an up-quark mass matrix [6, 7]. This is a significant feature of the yukawaon model.

The original motivation of the yukawaon model was to derive a charged lepton mass relation [8, 9, 10, 11]. For example, in the charged lepton sector, we assume

$$W_e = \lambda_e \text{Tr}[\Phi_e \Phi_e \Theta_e] + \mu_e \text{Tr}[Y_e \Theta_e] + W_\Phi, \quad (1.7)$$

where the fields  $\Phi_e$  and  $\Theta_e$  have the  $U(1)_X$  charges  $\frac{1}{2}x_e$  and  $-x_e$ , respectively. The term  $W_\Phi$  in the superpotential (1.7) has been introduced in order to fix a VEV spectrum of  $\langle \Phi \rangle$ . A SUSY vacuum condition  $\partial W / \partial \Theta_e = 0$  leads to a bilinear mass relation

$$\langle Y_e \rangle = -\frac{\lambda_e}{\mu_e} \langle \Phi_e \rangle \langle \Phi_e \rangle. \quad (1.8)$$

Therefore, the mass spectrum of the charged leptons are described by [12, 13]

$$K_e \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{v_1^2 + v_2^2 + v_3^2}{(v_1 + v_2 + v_3)^2} = \frac{\text{Tr}[\langle \Phi_e \rangle \langle \Phi_e \rangle]}{\text{Tr}^2[\langle \Phi_e \rangle]}, \quad (1.9)$$

and

$$\kappa_e \equiv \frac{\sqrt{m_e m_\mu m_\tau}}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^3} = \frac{v_1 v_2 v_3}{(v_1 + v_2 + v_3)^3} = \frac{\det \langle \Phi_e \rangle}{\text{Tr}^3[\langle \Phi_e \rangle]}, \quad (1.10)$$

where  $\langle \Phi_e \rangle = \text{diag}(v_1, v_2, v_3)$ . We refer the field  $\Phi_e$  as an “ur-yukawaon”. Although the mass relation (1.9) with  $K_e = 2/3$  is excellently satisfied with the observed charged lepton masses

(pole masses), it should be noted that masses which we deal with are not “pole” masses, but “running” masses. In the yukawaon model, the yukawaons  $Y_f$  (and also ur-yukawaon  $\Phi_e$ ) have their VEVs  $\langle Y_f \rangle$  (and  $\langle \Phi_e \rangle$ ) at a high energy scale  $\mu = \Lambda$ . In other words, the flavor symmetry  $O(3)$  is completely broken at  $\mu = \Lambda$ . Therefore, VEV relations which we obtain from SUSY vacuum conditions are valid only at  $\mu = \Lambda$ . The effective coupling constants  $Y_f^{eff} = (y_f/\Lambda)\langle Y_f \rangle$  evolve as in the standard model below the scale  $\Lambda$ . [The evolution of the relation (1.9) is given, for example, in Ref.[14, 15].] A naive estimate of  $\Lambda$  gives  $m_\nu \sim \langle H_u^0 \rangle^2/\Lambda$ , i.e.

$$\Lambda \sim 10^{12} \text{ GeV}, \quad (1.11)$$

where we have considered that all VEV values (except for  $\langle H_u^0 \rangle$  and  $\langle H_d^0 \rangle$ ) and coefficients  $\mu_f$  with a dimension of mass are of the order of  $\Lambda$ . Therefore, in the yukawaon model, what we should derive is not  $K_e(\Lambda) = 2/3$ , but  $K_e(\Lambda) = (2/3)(1 + \varepsilon)$  with  $\varepsilon \sim 10^{-3}$  [13]. However, the purpose of the present paper is not to discuss the value  $K_e(\Lambda)$ , so that we do not discuss a reasonable form of  $W_\Phi$ .

In the present paper, we are interested in massless scalars which appear when a flavor symmetry is spontaneously broken. Usually, when a flavor symmetry is spontaneously broken, the minimum number of the massless scalars is given by the Goldstone theorem. However, in the present yukawaon scenario, there are many yukawaons (and ur-yukawaons). [For example, even in the charged lepton sector, we need, at least, three  $3 \times 3$  scalars,  $Y_e$ ,  $\Phi_e$  and  $\Theta_e$  as seen in the superpotential (1.7).] It is a great concern to know whose components actually become massless, because this is important to discuss its physical meaning of the massless (and light) yukawaons. Regrettably, at present, the whole scenario of the yukawaon model is still not completed. Therefore, in this paper, we investigate only massless scalars in the charged lepton sector, because the sector is a pivotal point of the yukawaon model and the results are easily extended to other sectors.

In order to calculate masses of the yukawaons, we must know an explicit expression of the superpotential term  $W_\Phi$ . In this paper, we assume a form

$$W_e = \lambda \text{Tr}[\Phi_e \Phi_e \Theta_e] + \mu \text{Tr}[Y_e \Theta_e] + \varepsilon_{SB} \left\{ \lambda' \text{Tr}[\hat{\Phi}_e \Phi_e \Phi_e] + \lambda'' \text{Tr}[\hat{\Phi}_e \Phi_e Y_e] \right\}, \quad (1.12)$$

where  $\hat{\Phi}_e$  is a part of **5** of  $O(3)$  ( $\mathbf{3} \times \mathbf{3}$ )<sub>S</sub> and defined by  $\hat{\Phi}_e = \Phi_e - \frac{1}{3} \text{Tr}[\Phi_e]$ . Here, we have assigned  $U(1)_X$  charges as  $Q_X(Y_e) = x_e$ ,  $Q_X(\Phi_e) = \frac{1}{2}x_e$  and  $Q_X(\Theta_e) = -x_e$ , so that the  $U(1)_X$  symmetry is explicitly broken by the  $\varepsilon_{SB}$ -terms. We assume that the value  $\varepsilon_{SB}$  is negligibly small. Since we assign the  $R$  charges of  $\Phi_e$ ,  $Y_e$  and  $\Theta_e$  to 0, 0 and 2, respectively, the  $\varepsilon_{SB}$ -terms also break the  $R$  symmetry, so that the model (1.12) cannot break the supersymmetry spontaneously [16]. For the moment, we consider that the supersymmetry is unbroken, at least, in the Yukawaon sector.

Although the model (1.12) cannot give a realistic charged lepton mass spectrum at  $\mu = \Lambda$  as seen in Appendix, however, this is not essential problem for the present purpose to estimate massless (and light) yukawaons. If we consider an additional term to the superpotential (1.12) in order to obtain a reasonable mess spectrum (for example, see Ref.[13]), the calculation of the yukawaon masses becomes somewhat complicated, but we can obtain the same conclusions as

those which we obtain in the next section. For simplicity, in the present paper, we will adopt the superpotential form (1.12) as a toy model in the yukawaon model.

In the next section, we calculate the masses of  $Y_e$ ,  $\Phi_e$  and  $\Theta_e$  based on the yukawaon model (1.12) in the charged lepton sector. We will conclude that massless scalars are only three components  $(Y'_{12}, Y'_{23}, Y'_{31})$  of a linear combination  $Y'$  of the scalars  $Y_e$ ,  $\Phi_e$  and  $\Theta_e$ . We will also conclude that the components  $(Y'_{e11}, Y'_{e22}, Y'_{e33})$  have masses of the order of  $\varepsilon_{SB}\Lambda$  and the remaining components have masses of the order of  $\Lambda$ . In the section 3, we estimate contributions from yukawaons in the quark sector, and we check whether the conclusions only from the lepton sector are modified or not. Finally, the section 4 is devoted to concluding remarks (phenomenological effects of the massless yukawaons, a value of the scale  $\Lambda$ , and so on).

## 2 Massless yukawaons

Our interest is in explicit configurations of massless scalars in the present model (1.12), in which we have three  $3 \times 3$  scalars  $Y_e$ ,  $\Phi_e$  and  $\Theta_e$ . Since we consider an unbroken SUSY scenario at present, we calculate fermion masses instead of boson masses. The mass terms is obtained from the superpotential form (1.12) as follows:

$$\begin{aligned}
H_{mass} &= \lambda \text{Tr}[\langle \Theta \rangle \Phi \Phi] + \lambda \text{Tr}[\langle \Phi \rangle (\Phi \Theta + \Theta \Phi)] + \mu \text{Tr}[Y \Theta] \\
&+ \varepsilon_{SB} \lambda' \left( 3 \text{Tr}[\langle \Phi \rangle \Phi \Phi] - \frac{1}{3} \text{Tr}[\langle \Phi \rangle] \text{Tr}[\Phi \Phi] - \frac{2}{3} \text{Tr}[\langle \Phi \rangle \Phi] \text{Tr}[\Phi] \right) \\
&+ \varepsilon_{SB} \lambda'' \left( \text{Tr}[\langle Y \rangle \Phi \Phi] + \text{Tr}[\langle \Phi \rangle (\Phi Y + Y \Phi)] - \frac{1}{3} \text{Tr}[\langle \Phi \rangle] \text{Tr}[\Phi Y] - \frac{1}{3} \text{Tr}[\langle \Phi \rangle Y] \text{Tr}[\Phi] - \frac{1}{3} \text{Tr}[\langle Y \rangle \Phi] \text{Tr}[\Phi] \right) \\
&\equiv \text{Tr}[M_{\Phi\Phi} \Phi \Phi] + \text{Tr}[M_{\Phi Y} (\Phi Y + Y \Phi)] + \text{Tr}[M_{\Phi\Theta} (\Phi \Theta + \Theta \Phi)] + \text{Tr}[M_{\Theta Y} (\Theta Y + Y \Theta)] \\
&\quad + \text{Tr}[M_{\Phi} \Phi + M_Y Y] \text{Tr}[\Phi], \tag{2.1}
\end{aligned}$$

where we have dropped index “ $e$ ” for simplicity, and

$$\begin{aligned}
M_{\Phi\Phi} &= -\varepsilon_{SB} \frac{\lambda'' \lambda}{\mu} \left\{ 2 \langle \Phi \rangle \langle \Phi \rangle - \frac{1}{3} \text{Tr}[\langle \Phi \rangle] \langle \Phi \rangle - \xi \left( 3 \text{Tr}[\langle \Phi \rangle] \langle \Phi \rangle - \frac{1}{3} \text{Tr}^2[\langle \Phi \rangle] \mathbf{1} \right) \right\}, \\
M_{\Phi Y} &= \varepsilon_{SB} \lambda'' \left( \langle \Phi \rangle - \frac{1}{6} \text{Tr}[\langle \Phi \rangle] \mathbf{1} \right), \quad M_{\Theta Y} = \frac{1}{2} \mu \mathbf{1}, \\
M_{\Phi} &= -\frac{2}{3} \varepsilon_{SB} \lambda' \langle \Phi \rangle + \frac{1}{3} \varepsilon_{SB} \frac{\lambda'' \lambda}{\mu} \langle \Phi \rangle \langle \Phi \rangle, \quad M_Y = -\frac{1}{3} \varepsilon_{SB} \lambda'' \langle \Phi \rangle. \tag{2.2}
\end{aligned}$$

We calculate the mass matrix on the basis in which  $\langle \Phi \rangle$  is diagonal, i.e.  $\langle \Phi \rangle = \text{diag}(v_1, v_2, v_3)$ .

When we define  $\Psi^T = (Y, \Phi, \Theta)$ , we obtain a mass matrix for the components  $\Psi_{ij}$  ( $i \neq j$ ) as follows:

$$H_{mass} = \frac{1}{2} \sum_{i \neq j} \Psi_{ij}^T M_{(ij)} \Psi_{ij}, \tag{2.3}$$

$$M_{(ij)} = \begin{pmatrix} 0 & M_{\Phi Y}^{(ij)} & M_{\Theta Y}^{(ij)} \\ M_{\Phi Y}^{(ij)} & M_{\Phi\Phi}^{(ij)} & M_{\Phi\Theta}^{(ij)} \\ M_{\Theta Y}^{(ij)} & M_{\Phi\Theta}^{(ij)} & 0 \end{pmatrix}, \quad (2.4)$$

where

$$\begin{aligned} M_{\Phi\Phi}^{(ij)} &= (M_{\Phi\Phi})_{ii} + (M_{\Phi\Phi})_{jj} = -\varepsilon_{SB} \frac{\lambda''\lambda}{\mu} \left\{ 2(v_i^2 + v_j^2) \right. \\ &\quad \left. - \frac{1}{3}(v_i + v_j) \sum_k v_k - \xi \left( 3(v_i + v_j) - \frac{2}{3}\text{Tr}[\langle\Phi\rangle] \right) \text{Tr}[\langle\Phi\rangle] \right\}, \\ M_{\Phi Y}^{(ij)} &= (M_{\Phi Y})_{ii} + (M_{\Phi Y})_{jj} = \varepsilon_{SB} \lambda'' \left( v_i + v_j - \frac{1}{3}\text{Tr}[\langle\Phi\rangle] \right), \\ M_{\Phi\Theta}^{(ij)} &= (M_{\Phi\Theta})_{ii} + (M_{\Phi\Theta})_{jj} = \lambda(v_i + v_j), \\ M_{\Theta Y}^{(ij)} &= (M_{\Theta Y})_{ii} + (M_{\Theta Y})_{jj} = \mu. \end{aligned} \quad (2.5)$$

Since there is no mixing term between  $\Psi_{ij}$  and  $\Psi_{kk}$ , the mass matrix (2.4) is substantially a  $3 \times 3$  matrix. In order to see whether there is a massless state or not, we calculate  $\det M_{(ij)}$ :

$$\begin{aligned} \det M_{(ij)} &= 2M_{\Phi Y}^{(ij)} M_{\Phi\Theta}^{(ij)} M_{\Theta Y}^{(ij)} - M_{\Phi\Phi}^{(ij)} (M_{\Theta Y}^{(ij)})^2 \\ &= \varepsilon_{SB} \lambda'' \lambda \mu \left\{ 2(v_i + v_j)^2 + 2(v_i^2 + v_j^2) - (v_i + v_j) \text{Tr}[\langle\Phi\rangle] - \xi \left( 3(v_i + v_j) - \frac{2}{3}\text{Tr}[\langle\Phi\rangle] \right) \text{Tr}[\langle\Phi\rangle] \right\}. \end{aligned} \quad (2.6)$$

In this model, we must take  $\xi = 1$  (see Eq.(A.13) in Appendix), so that we obtain

$$\det M_{(ij)} = \frac{2}{3} \varepsilon_{SB} \lambda'' \lambda \mu \left\{ v_i^2 + v_j^2 + v_k^2 - 4(v_i v_j + v_j v_k + v_k v_i) \right\}, \quad (2.7)$$

where we have used  $[\langle\Phi\rangle] = \sum_k v_k = v_i + v_j + v_k$ . In the present model, as shown in (A.14) in Appendix, the VEV values  $v_i$  satisfy the relation  $v_1^2 + v_2^2 + v_3^2 = (2/3)(v_1 + v_2 + v_3)^2$ , i.e.  $v_1^2 + v_2^2 + v_3^2 = 4(v_1 v_2 + v_2 v_3 + v_3 v_1)$ , so that the value of (2.7) becomes exactly zero. We can see that there are no massless states more than three by calculating  $\text{Tr}[M_{(ij)}]$  and  $\text{Tr}[M_{(ij)}^2]$ .

More simply, we can demonstrate it by seeing the case of  $\varepsilon_{SB} \rightarrow 0$ . [The case  $\varepsilon_{SB} \rightarrow 0$  does not mean  $\varepsilon_{SB} = 0$ . The case  $\varepsilon_{SB} = 0$  means a trivial case with  $W_\Phi = 0$  in Eq.(1.3), so that  $\langle\Theta_e\rangle = \langle Y_e\rangle = \langle\Phi_e\rangle = 0$  or  $\langle\Theta_e\rangle = 0$  and  $\langle Y_e\rangle = -(\lambda_e/\mu_e)\langle\Phi_e\rangle\langle\Phi_e\rangle \neq 0$ . Here, the case  $\varepsilon_{SB} \rightarrow 0$  means a mass matrix  $M_{(ij)}$  with  $\varepsilon_{SB} \simeq 0$ .] In this case, the mass matrix  $M_{(ij)}$  is given by

$$M_{(ij)} = \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ a & b & 0 \end{pmatrix}, \quad (2.8)$$

where

$$a = \mu, \quad b = \lambda(v_i + v_j). \quad (2.9)$$

The eigenvalues and mixing matrix are given by

$$m(Y') = 0, \quad m(\Phi') = -\sqrt{a^2 + b^2}, \quad m(\Theta') = \sqrt{a^2 + b^2}, \quad (2.10)$$

$$U = \begin{pmatrix} c & \frac{1}{\sqrt{2}}s & \frac{1}{\sqrt{2}}s \\ -s & \frac{1}{\sqrt{2}}c & \frac{1}{\sqrt{2}}c \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (2.11)$$

respectively, where

$$s = \frac{a}{\sqrt{a^2 + b^2}}, \quad c = \frac{b}{\sqrt{a^2 + b^2}}, \quad (2.12)$$

$$\begin{pmatrix} Y \\ \Phi \\ \Theta \end{pmatrix} = U \begin{pmatrix} Y' \\ \Phi' \\ \Theta' \end{pmatrix}, \quad (2.13)$$

and  $(Y', \Phi', \Theta')$  are mass-eigenstates. As seen in Eqs.(2.10) and (2.11), the massless states are only three, i.e.  $(Y'_{12}, Y'_{23}, Y'_{31})$ , and they couple to the charged lepton sector as

$$\frac{y_e}{\Lambda} \frac{b}{\sqrt{a^2 + b^2}} \sum_{i \neq j} \ell_i Y'_{ij} e_j^c H_d. \quad (2.14)$$

The situation is almost unchanged for the case  $\varepsilon_{SB} \neq 0$  (but  $\varepsilon_{SB} \simeq 0$ ). Since we consider  $\Lambda \sim 10^{12}$  GeV (for details, see the final section 4), the effective coupling constant of  $e_i Y'_{ij} e_j^c$  is of the order of  $\langle H_d \rangle / \Lambda \sim 10^{-13}$  (for  $\tan \beta \sim 10$ ). The phenomenological meaning will be discussed in the final section 4.

Next, we calculate a mass matrix for the components  $\Psi_{ii}$ . Since there are mixing terms between  $\Psi_{ii}$  and  $\Psi_{jj}$ , i.e.  $\text{Tr}[M_\Phi \Phi + M_Y Y] \text{Tr}[\Phi]$ , the mass matrix is described by  $9 \times 9$  matrix for  $\Psi_{ii} = (Y_{11}, Y_{22}, Y_{33}, \Phi_{11}, \Phi_{22}, \Phi_{33}, \Theta_{11}, \Theta_{22}, \Theta_{33})$ :

$$M_{(ii)} = \begin{pmatrix} 0 & A & C \\ A & B & D \\ C & D & 0 \end{pmatrix}, \quad (2.15)$$

where  $C$  and  $D$  take diagonal forms

$$C = \frac{1}{2} \mu \mathbf{1}, \quad D = \lambda \text{diag}(v_1, v_2, v_3), \quad (2.16)$$

while  $A$  and  $B$  have off-diagonal elements as

$$A_{ii,ii} = \varepsilon_{SB} \lambda'' \left( v_i - \frac{1}{6} \text{Tr}[\langle \Phi \rangle] \right), \quad A_{ii,jj} = -\frac{1}{6} \varepsilon_{SB} \lambda'' (v_i + v_j), \quad (2.17)$$

$$B_{ii,ii} = -\varepsilon_{SB} \frac{\lambda'' \lambda}{\mu} \left( 2v_i^2 - \frac{1}{3} v_i \text{Tr}[\langle \Phi \rangle] - 3\xi v_i \sum_k v_k + \frac{1}{3} \xi \text{Tr}[\langle \Phi \rangle]^2 \right),$$

$$B_{ii,jj} = \frac{1}{6} \varepsilon_{SB} \frac{\lambda'' \lambda}{\mu} (v_i^2 + v_j^2 - 2\xi(v_i + v_j) \text{Tr}[\langle \Phi \rangle]). \quad (2.18)$$

In order to see whether there are massless states or not, we calculate  $\det M_{(ii)}$ :

$$\det M_{(ii)} = \frac{1}{2(24)^3} (\varepsilon_{SB} \lambda'' \lambda \mu \text{Tr}^2[\langle \Phi \rangle])^3 (-171K^3 - 1053K^2 + 2283K - 947$$

$$+ 93092\kappa^2 - 17526\kappa + 32846\kappa K), \quad (2.19)$$

for  $\xi = 1$ . For the value  $K = 2/3$ , we obtain

$$\det M_{(ii)} = \frac{1}{6(24)^3} (\varepsilon_{SB} \lambda'' \lambda \mu \text{Tr}^2[\Phi])^3 (169 + 1114\kappa + 372368\kappa^2), \quad (2.20)$$

so that the value (2.20) can never become zero either for a positive  $\kappa$  [see (A.17) in Appendix] or for the negative value  $\kappa = -1/54$  given in Eq.(A.15). Therefore, we cannot have massless scalars for the components  $\Psi_{ii}$ .

However, in the limit of  $\varepsilon_{SB} \rightarrow 0$ , the mass matrix  $M_{(ii)}$  becomes the same type with (2.8), so that  $Y'_{(ii)}$  become massless. Therefore, by taking the result (2.20) into consideration, we can conclude that the masses of  $Y'_{ii}$  are given by

$$m(Y'_{ii}) \sim \varepsilon_{SB} \lambda'' \lambda \mu \frac{\text{Tr}^2[\langle \Phi \rangle]}{\Lambda^2} \sim \varepsilon_{SB} \Lambda. \quad (2.21)$$

If we consider  $\varepsilon_{SB} \sim 10^{-12}$ , it is possible that the masses of  $Y'_{ii}$  appear in a TeV region. However, at present, we do not fix the order of  $\varepsilon_{SB}$ .

In conclusion, we have strictly calculated masses of yukawaons in the charged lepton sector based on a superpotential form (1.12). We have found that, of the fields  $Y_e$ ,  $\Phi_e$  and  $\Theta_e$ , the massless components are only three ( $Y'_{e12}, Y'_{e23}, Y'_{e31}$ ). The components ( $Y'_{e11}, Y'_{e22}, Y'_{e33}$ ) have masses of the order of  $\varepsilon_{SB} \Lambda$  and the remaining components have masses of the order of  $\Lambda$ .

### 3 Contribution from quark sector

As far as we see the results in the charged lepton sector, there is no mixing between different components  $\Psi_{ij}$  and  $\Psi_{kl}$  (but there are mixings between  $\Psi_{ii}$  and  $\Psi_{jj}$ ). However, this is not true

when we consider whole yukawaons in the all sectors. For example, in order to give a nearly tribimaximal mixing [17, 18, 19, 20, 21, 22], the author [6] has proposed superpotential terms

$$W_R = \mu_R \text{Tr}[Y_R \Theta_R] + \lambda_R \text{Tr}[(\Phi_u Y_e + Y_e \Phi_u) \Theta_R]. \quad (3.1)$$

where  $\langle \Phi_u \rangle$  is related to up-quark mass matrix  $M_u$  via a relation

$$\langle Y_u \rangle = -\frac{\lambda_u}{\mu_u} \langle \Phi_u \rangle \langle \Phi_u \rangle. \quad (3.2)$$

from superpotential terms in up-quark sector

$$W_u = \lambda_u \text{Tr}[\Phi_u \Phi_u \Theta_u] + \mu_u \text{Tr}[Y_u \Theta_u], \quad (3.3)$$

similar to Eq.(1.7) (neglecting  $W_{\Phi_u}$ ). Then, the terms (3.1) contribute to the yukawaon mass terms as

$$\lambda_R \text{Tr}[\langle \Theta_R \rangle (Y_e \Phi_u + \Phi_u Y_e)] + \lambda_R \text{Tr}[\langle \Phi_u \rangle (Y_e \Theta_R + \Theta_R Y_e)]. \quad (3.4)$$

In this case,  $\langle \Theta_R \rangle$  is zero [6], but  $\langle \Phi_u \rangle$  is not zero, and besides,  $\langle \Phi_u \rangle$  is not diagonal on the diagonal basis of  $\langle \Phi_e \rangle$ . The existence of the terms (3.1) causes  $Y'_{ij}$ - $Y'_{jk}$  mixings among the massless yukawaon  $Y'_e$  described in Eq.(2.10). In other words, the flavor-changing neutral currents (FCNC) via  $Y'_{ij}$  can appear in principle. From superpotential terms in the up-quark sector From the superpotential (3.3), we also obtain mass terms

$$\lambda_u \text{Tr}[\langle \Theta_u \rangle \Phi_u \Phi_u + \langle \Phi_u \rangle (\Phi_u \Theta_u + \Theta_u \Phi_u)] + \mu_u \text{Tr}[Y_u \Theta_u]. \quad (3.5)$$

As a result, we obtain the following mass matrix for  $\Psi^{(u)} = (Y_u, \Phi_u, \Theta_u, Y_e, Y_R, \Theta_R)$

$$M^{(u)} = \begin{pmatrix} 0 & 0 & d_u & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 & 0 \\ d_u & a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c \\ 0 & 0 & 0 & 0 & 0 & d_R \\ 0 & b & 0 & c & d_R & 0 \end{pmatrix}, \quad (3.6)$$

where

$$a = \lambda_u \langle \Phi_u \rangle, \quad b = \lambda_R \langle Y_e \rangle, \quad c = \lambda_R \langle \Phi_u \rangle, \quad d_u = \frac{1}{2} \mu_u \mathbf{1}, \quad d_R = \frac{1}{2} \mu_R \mathbf{1}, \quad (3.7)$$

and we have put  $\langle \Theta_u \rangle = \langle \Theta_R \rangle = 0$ . Although the matrix  $b$  is not diagonal on the diagonal basis of  $\langle \Phi_u \rangle$ , in order to obtain a rough sketch of the mixing, we regard the matrix (3.7) as a  $6 \times 6$  matrix. Then, we obtain two massless states  $Y'_u$  and  $Y'_e$ , which have the following components

$$Y'_u = \frac{ad_R}{N_u} Y_u - \frac{bd_R}{N_u} \Phi_u + \frac{bd_u}{N_u} Y_R, \quad (3.8)$$

$$Y'_e = \frac{ac}{N_e} Y_u - \frac{cd_u}{N_e} \Phi_u + \frac{bd_u}{N_e} Y_e, \quad (3.9)$$



respectively, where  $N_u = \sqrt{(ad_R)^2 + (bd_R)^2 + (bd_u)^2}$  and  $N_e = \sqrt{(ac)^2 + (cd_u)^2 + (bd_u)^2}$ . The state  $Y'_e$  in (3.9) is not identical with  $Y'_e$  given in (2.13). The former is defined as a massless state in the mass matrix for  $\Psi^{(u)} = (Y_u, \Phi_u, \Theta_u, Y_e, Y_R, \Theta_R)$ , and the latter is defined as a massless state in the mass matrix for  $\Psi = (Y_e, \Phi_e, \Theta_e)$ . Of course, the exact mass-eigenstates should be calculated by diagonalizing a mass matrix for whole yukawaons simultaneously. Nevertheless, from the results (3.8) and (3.9), we can deduce the following overviews: (i) The massless particle  $Y'_u$  contains a component  $Y_u$ , but it does not contain  $Y_e$ , while the massless particle  $Y'_e$  contains both components  $Y_u$  and  $Y_e$ . (ii) We consider that  $Y'_u$  has a mass of the order  $\varepsilon_{SB}\Lambda$  due to terms which we have neglected in (3.2) (which is corresponding to  $W_{\Phi_u}$ ), but  $Y'_e$  will be still massless, so that only three massless components ( $Y'_{12}, Y'_{23}, Y'_{31}$ ) can couple to all quark and lepton sectors. Here, we have dropped the index “e” from the latter particles  $Y'_e$  because the particles  $(Y'_e)_{ij}$  couple not only to the charged lepton sector but also to whole quark and lepton sectors (except for the  $\nu_i^c \nu_j^c$  sector). (iii) Since the VEV matrices  $\langle Y_u \rangle$  and  $\langle Y_e \rangle$  cannot simultaneously be diagonalized, the indexes (1, 2, 3) in  $(Y'_{12}, Y'_{23}, Y'_{31})$  do not mean either  $(1, 2, 3) = (e, \mu, \tau)$  or  $(1, 2, 3) = (u, c, t)$ . Therefore, FCNC effects can appear via exchanges of  $Y'$  in principle.

#### 4 Concluding remarks

In conclusion, we have estimate yukawaon masses from the superpotential terms (1.12) and (3.1) explicitly, and we have obtained three massless yukawaons ( $Y'_{12}, Y'_{23}, Y'_{31}$ ) and three light yukawaons ( $Y'_{11}, Y'_{22}, Y'_{33}$ ). The superpotential term is only a toy model, the results seem to be general ones. Finally, we would like to check whether those particles are harmless or not in phenomenology (except for cosmological problems).

For example, massless yukawaon  $Y'_{12}$  couple to charged lepton sector, so that it causes a muon decay  $\mu \rightarrow e + Y'_{12}$ . However, it is invisibly small compared with the weak decay  $\mu \rightarrow e + \bar{\nu} + \nu$ , because the effective coupling constant of the term  $\ell Y_e e^c$  is given by

$$g \equiv y_e \frac{\langle H_d \rangle}{\Lambda} \sim 10^{-11}, \quad (4.1)$$

where we have taken  $v_d \equiv \langle H_d \rangle \sim 10^1$  GeV (for  $\tan \beta \sim 10$ ) and  $\Lambda \sim 10^{12}$  GeV. Similarly, the massless yukawaons ( $Y'_{12}, Y'_{23}, Y'_{31}$ ) can couple not only to the charged lepton sector but also to other quark and lepton sectors. However, production rates of  $Y'_{ij}$  and decay rates of the conventional quarks and leptons into  $Y'_{ij}$  are invisibly small because of the too small effective coupling constant (4.1).

By the way, can we regard the light yukawaons ( $Y'_{11}, Y'_{22}, Y'_{33}$ ) with masses of the order of  $\varepsilon_{SB}\Lambda$  as a candidate of the dark matter? Since we have interaction terms  $\varepsilon_{SB}\lambda' \text{Tr}[\hat{\Phi}_e \Phi_e \Phi_e] + \varepsilon_{SB}\lambda'' \text{Tr}[\hat{\Phi}_e \Phi_e Y_e]$  in Eq.(1.12), we can have physical interaction terms  $Y'_{ij} Y'_{jk} Y'_{ki}$ , so that the light yukawaons  $Y'_{ii}$  can decay into  $\tilde{Y}'_{ij} + \tilde{Y}'_{ij}$  with the effective coupling constant of the order  $\varepsilon_{SB}$ , where  $\tilde{Y}'$  denotes a SUSY partner of the scalar  $Y'$ . (Although we have interactions  $\lambda \text{Tr}[\Phi_e \Phi_e \Theta_e]$  without the factor  $\varepsilon_{SB}$  in Eq.(1.12), the field  $\Theta_e$  does not contain either the massless yukawaons or the light yukawaons, the effective coupling constants of  $Y' Y' Y'$  cannot be given by  $\lambda$  without the factor  $\varepsilon_{SB}$ .) Since  $Y'_{ii}$  have masses  $m_{Y'}$  of the order of  $\varepsilon_{SB}\Lambda$  as seen in Eq.(2.21), the decay widths are of the order of  $\varepsilon_{SB}^2 m_{Y'} \sim \varepsilon_{SB}^3 \Lambda$ . At present, the parameter  $\varepsilon_{SB}$  is free. Therefore,

the lifetimes can freely be adjusted. However, regrettably, we cannot regard the light yukawaons as candidates of the cold dark matter: In order to get a small decay width of the order  $10^{-43}$  GeV which is required from the age of the universe, we must consider

$$10^{-45} \text{ GeV} \sim \Gamma(Y'_{ii}) \sim \frac{(\varepsilon_{SB})^2}{4\pi} m(Y'_{ii}) \sim 10^{-1} (\varepsilon_{SB})^3 \Lambda, \quad (4.2)$$

which leads to  $\varepsilon_{SB} \sim 10^{-18}$  for  $\Lambda \sim 10^{12}$  GeV, so that we get  $m(Y'_{ii}) \sim 10^{-6}$  GeV. This value of  $m(Y'_{ii})$  is too small to regard  $Y'_{ii}$  as a candidate of the cold dark matter.

However, here, let us recheck the estimate of  $\Lambda$ , (1.11). The result  $\Lambda \sim 10^{12}$  GeV is obtained as follows: From the relation (1.1) in the charged lepton sector, we estimate

$$\frac{\langle Y_e \rangle}{\Lambda} \sim \frac{1 \text{ GeV}}{y_e v_d} \sim \frac{1}{y_e} 10^{-1}, \quad (4.3)$$

where we have assumed  $\tan \beta \equiv v_u/v_d \sim 10$ , so that  $v_d \equiv \langle H_d \rangle \sim 10$  GeV. If we consider  $y_e \sim 10^{-1}$ , we can consider  $\langle Y_e \rangle \sim \Lambda$ . On the other hand, from the seesaw neutrino mass matrix  $M_\nu = m_D M_R^{-1} m_D^T$ , where  $m_D = (y_\nu/\Lambda) \langle Y_\nu \rangle v_u$  and  $M_R = y_R \langle Y_R \rangle \sim \Lambda_R$ , we estimate

$$\Lambda_R \sim \frac{(1 \text{ GeV})^2}{10^{-10} \text{ GeV}} \left( \frac{v_u}{v_d} \right)^2 \left( \frac{y_\nu}{y_e} \right)^2 \sim 10^{12} \text{ GeV}, \quad (4.4)$$

where we have taken  $m_{\nu 3} \sim 10^{-10}$  GeV and  $(y_\nu/y_e)^2 \sim 1$ . (In a neutrino model given in Ref.[23], the  $U(1)_X$  charges of  $e^c$  and  $\nu^c$  are the same, so that the yukawaon  $Y_e$  couples not only to the charged lepton sector, but also to the neutrino sector, and we can consider a model without  $Y_\nu$ .) Note that the scale  $\Lambda$  is independent of the scale  $\Lambda_R$  at present.

The relation between  $\Lambda$  and  $\Lambda_R$  comes from the relation (3.1):

$$\Lambda_R \sim \frac{1}{\mu_R} \langle \Phi_u \rangle \langle Y_e \rangle \sim \frac{\Lambda^2}{\mu_R}. \quad (4.5)$$

The simplest interpretation of (4.5) is to consider

$$\Lambda_R \sim \Lambda \sim \mu_R \sim 10^{12} \text{ GeV}. \quad (4.6)$$

Thus, the conclusion (1.11) has been obtained.

However, in a recent version of the yukawaon model in the neutrino sector, the author [7] has proposed a modified superpotential terms

$$W_R = \mu_R \text{Tr}[Y_R \Theta_R] + \frac{\lambda_R}{\Lambda} \text{Tr}[(\Phi_u P_u Y_e + Y_e P_u \Phi_u) \Theta_R], \quad (4.7)$$

where  $P_u$  is a new field which appears only in the right-handed neutrino sector. If we regard the order of  $\langle P_u \rangle$  as  $\langle P_u \rangle \sim \Lambda_R$ , the relation (4.5) is replaced with

$$\Lambda_R \sim \frac{1}{\mu_R \Lambda} \langle \Phi_u \rangle \langle Y_e \rangle \langle P_u \rangle \sim \frac{\Lambda^2 \Lambda_R}{\mu_R \Lambda}. \quad (4.8)$$

Then, if we regarded  $\mu_R$  as  $\mu_R \sim \Lambda$ , the scale  $\Lambda$  can be independent of the scale  $\Lambda_R$ , so that we can take a lower value than  $\Lambda_R \sim 10^{12}$  GeV. Of course, a scale of  $\Lambda$  with too low value is dangerous phenomenologically. A case with a lower value  $\Lambda$  will be investigated elsewhere.

Thus, we can conclude that the massless and light yukawaons are harmless in low and high energy physics, as far as we consider  $\Lambda \sim 10^{12}$  GeV. In other words, regrettably, it is hard to find a positive effect of these yukawaons in low and high energy physics experiments. On the other hand, in cosmology, it is likely that the existence of such massless yukawaons causes problems. (Besides, there is a possibility that the massless yukawaons acquire unwelcome masses due to radiative corrections.) If such troubles turn out to be serious, we will be obliged to consider the flavor symmetry as a gauged one.

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### Appendix: VEV structure of the ur-yukawaon

In this appendix, we discuss a VEV structure of the superpotential (1.12), i.e.

$$\begin{aligned} W_e = & \lambda \text{Tr}[\Phi_e \Phi_e \Theta_e] + \mu \text{Tr}[Y_e \Theta_e] + \varepsilon_{SB} \lambda' \left( \text{Tr}[\Phi_e \Phi_e \Phi_e] - \frac{1}{3} \text{Tr}[\Phi_e] \text{Tr}[\Phi_e \Phi_e] \right) \\ & + \varepsilon_{SB} \lambda'' \left( \text{Tr}[\Phi_e \Phi_e Y_e] - \frac{1}{3} \text{Tr}[\Phi_e] \text{Tr}[\Phi_e Y_e] \right). \end{aligned} \quad (A.1)$$

Here, we have assumed that the  $U(1)_X$  symmetry is explicitly broken by the order  $\varepsilon_{SB}$  which is negligibly small.

SUSY vacuum conditions  $\partial W / \partial Y_e = 0$  and  $\partial W / \partial \Phi_e = 0$  lead to

$$\Theta_e = -\varepsilon_{SB} \frac{\lambda''}{\mu} \hat{\Phi}_e \Phi_e, \quad (A.2)$$

and

$$\begin{aligned} \frac{\partial W}{\partial \Phi_e} = & \lambda(\Phi_e \Theta_e + \Theta_e \Phi_e) + \varepsilon_{SB} \lambda' \left( 3\Phi_e^2 - \frac{2}{3} \text{Tr}[\Phi_e] \Phi_e - \frac{1}{3} \text{Tr}[\Phi_e \Phi_e] \mathbf{1} \right) \\ & + \varepsilon_{SB} \lambda'' \left( \Phi_e Y_e + Y_e \Phi_e - \frac{1}{3} \text{Tr}[\Phi_e] Y_e - \frac{1}{3} \text{Tr}[\Phi_e Y_e] \mathbf{1} \right) = 0, \end{aligned} \quad (A.3)$$

respectively. By substituting (A.2) into (A.3), we obtain a cubic equation for  $\langle \Phi_e \rangle$ :

$$c_3 \langle \Phi_e \rangle^3 + c_2 \langle \Phi_e \rangle^2 + c_1 \langle \Phi_e \rangle + c_0 \mathbf{1} = 0. \quad (A.4)$$

where

$$c_3 = 4, \quad c_2 = -(1 + 3\xi)\text{Tr}[\Phi_e], \quad c_1 = \frac{2}{3}\xi\text{Tr}^2[\Phi_e], \quad c_0 = -\frac{1}{3}(\text{Tr}[\Phi_e\Phi_e\Phi_e] - \xi\text{Tr}[\Phi_e]\text{Tr}[\Phi_e\Phi_e]), \quad (\text{A.5})$$

and  $\xi$  is defined by

$$\xi = \frac{\lambda'}{\lambda''} \frac{\mu}{\lambda\text{Tr}[\Phi_e]}. \quad (\text{A.6})$$

The coefficients  $c_a$  ( $a = 0, 1, 2, 3$ ), in general, have the following relations:

$$\frac{c_2}{c_3} = -\text{Tr}[\langle\Phi_e\rangle], \quad \frac{c_1}{c_3} = \frac{1}{2}(\text{Tr}^2[\langle\Phi_e\rangle] - \text{Tr}[\langle\Phi_e\rangle\langle\Phi_e\rangle]), \quad \frac{c_0}{c_3} = -\det\langle\Phi_e\rangle. \quad (\text{A.7})$$

Therefore, we can predict the ratios  $K_e$  and  $\kappa_e$  as follows:

$$K_e = 1 - 2\frac{c_1}{c_3} \frac{1}{\text{Tr}^2[\langle\Phi_e\rangle]}, \quad (\text{A.8})$$

$$\kappa_e = -\frac{c_0}{c_3} \frac{1}{\text{Tr}^3[\langle\Phi\rangle]}, \quad (\text{A.9})$$

by using the relations (A.7).

Note that the coefficients (A.4) are obtained independently of the value of  $\varepsilon_{SB}$ . By using a general formula for any  $3 \times 3$  matrix

$$L \equiv \frac{\text{Tr}[\Phi\Phi\Phi]}{\text{Tr}^3[\Phi]} = 3\kappa + \frac{3}{2}K - \frac{1}{2}, \quad (\text{A.10})$$

we can obtain

$$K_e \equiv \frac{\text{Tr}[\Phi_e\Phi_e]}{\text{Tr}^2[\Phi_e]} = 1 + 2\frac{c_1}{c_2} \frac{1}{\text{Tr}[\Phi_e]} = 1 - \frac{1}{3}\xi, \quad (\text{A.11})$$

$$\kappa_e \equiv \frac{\det\Phi_e}{\text{Tr}^3[\Phi_e]} = \frac{c_0}{c_2} \frac{1}{\text{Tr}^2[\Phi_e]} = \frac{1}{12}(L_e - \xi K_e) = \frac{1}{4}\left(\kappa_e + \frac{1}{2}K_e - \frac{1}{6} - \frac{1}{3}\xi K_e\right). \quad (\text{A.12})$$

On the other hand, in order to obtain a solution with  $[\Phi_e] \neq 0$ , from the relations (A.4) for the coefficients  $c_3$  and  $c_2$ , the parameter  $\xi$  must be taken as

$$\xi = 1. \quad (\text{A.13})$$

Then, we can obtain

$$K_e(\Lambda) = \frac{2}{3}, \quad (\text{A.14})$$

from Eq.(A.11). Note that the result  $K_e(\Lambda) = 2/3$  has been obtained independently of the parameter  $\varepsilon_{SB}$ . Again, we would like to emphasize that the result  $K_e(\mu) = 2/3$  is valid only at

$\mu = \Lambda$ , and it cannot explain the observed fact  $K_e^{pole} = 2/3$ . Therefore, the form of  $W_\Phi$  (1.12) is only a toy model in the yukawaon approach.

On the other hand, from Eq.(A.12), we obtain

$$\kappa_e = \frac{1}{3} \left\{ \left( \frac{1}{2} - \frac{1}{3}\xi \right) K_e - \frac{1}{6} \right\} = -\frac{1}{54}. \quad (A.15)$$

The observed value [24]

$$\kappa^{pole} = (2.0633 \pm 0.0001) \times 10^{-3}, \quad (A.16)$$

leads to [25]

$$\kappa_e(\mu) = (2.023 \pm 0.0001) \times 10^{-3}, \quad (A.17)$$

at  $\mu = 2 \times 10^{16}$  GeV for a SUSY model with  $\tan \beta = 10$ . Therefore, the toy model (1.12) cannot explain the observed value of  $\kappa$  at all.

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